

Annihilation Mechanisms of e^+e^- and $\mu^+\mu^-$ Production in Relativistic Nuclear Collisions

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Abstract. The $\pi^+\pi^-$ and $q\bar{q}$ annihilation mechanisms of dilepton production during relativistic nuclear collisions are studied. We focus on the modifications caused by the specific features of in-medium pion and quark states rather than by medium modification of the ρ -meson spectral density. The main ingredient emerging in our approach is a form-factor of the multi-pion (multi-quark) system. Replacing the usual delta-function the form-factor plays the role of distribution which, in some sense, "connects" the total 4-momenta of the annihilating and outgoing particles. The difference between the c.m.s. velocities attributed to annihilating and outgoing particles is a particular consequence of this replacement and results in the appearance of a new factor in the formula for the lepton pair production rate. We obtained that the form-factor of the multi-pion (multi-quark) system causes broadening of the rate which is most pronounced for small invariant masses, in particular, we obtain a growth of the rate for the invariant masses below two masses of the annihilating particles.

Keywords: Relativistic nuclear collisions; dilepton production; off-shell effects.

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1. Introduction

The main objective in studying strongly interacting matter, which is formed during relativistic heavy ion collisions, is identification of the quark-gluon plasma. Lepton pairs and photons created during relativistic nucleus-nucleus collision do not interact with the highly excited nuclear matter, they leave the reaction zone without further rescattering. That is why, the dileptons (e^+e^- and $\mu^+\mu^-$ pairs) observed in high-energy heavy ion collisions carry undistorted information on the dense early stage

of the reaction as well as on its dynamics. The enhancement with an invariant mass of $200 \div 800$ MeV observed by the CERES collaboration [1, 2] in the production of dileptons has received a considerable attention (for the review, see Ref. [3]). It was found that a large part of the observed enhancement is due to the medium effects (see Refs. [4, 5] and references therein). Meanwhile, pion annihilation is the main source of dileptons which come from the hadron matter [6, 7] (see also [8]). That is why, the proper analysis of the dilepton spectra obtained experimentally gives important data which probe the pion dynamics in the dense nuclear matter. The purpose of the present paper is to look once more on the $\pi^+\pi^-$ and $q\bar{q}$ annihilation mechanisms of dilepton production from the hadron and quark-gluon plasmas by accounting the medium-induced modifications of the dilepton spectrum. In order to do this, we concentrate on the modifications which are due rather to intramedium pion and quark states, than on the discussion of a modification of the ρ -meson spectral density. In accordance with our suggestions, the main features of a pion wave function follow from the fact that pions live a finite time in the system where they can take part in the annihilation reaction. As a consequence, the off-shell effects give an appreciable contribution to the features of the annihilation process specifically in the region of low invariant masses. Moreover, if pions are the entities of a local subsystem, then the spatial structure of the pion states is far from a plane-wave one and this also gives the essential contribution to the features of the dilepton spectrum.

2. Annihilation of particles in finite space-time volume

To carry out the outlined program, we assume that the pion liquid formed after the equilibration exists in a finite volume, and the confinement of pions to this volume is a direct consequence of the presence of the dense hadron environment which prevents the escape of pions during some mean lifetime τ . The same can be assumed concerning a hot system of quarks which are confined to a quark-gluon droplet. So, we assume the system of pions (quarks) produced in high-energy heavy-ion collisions is effectively bounded in a finite volume. The pion (quark) wave functions $\phi_\lambda(x)$, where λ is a quantum number, satisfy the proper boundary conditions and belong to the complete set of functions. For instance, the stationary wave functions may be taken as the solutions of the Klein-Gordon equation $(\nabla^2 + k^2)\phi_\lambda(\mathbf{x}) = 0$, where $k^2 = E^2 - m^2$, which satisfy the Dirichlet boundary condition on the surface S : $\phi_\lambda(\mathbf{x})|_S = 0$. For the box boundary, we get $\phi_{\mathbf{k}}(\mathbf{x}) = \sqrt{8/V} \prod_{i=1}^3 \theta(L_i - x_i) \theta(x_i) \sin(k_i x_i)$, where $V = L_1 L_2 L_3$ is the box volume, $\lambda \equiv \mathbf{k} = (k_1, k_2, k_3)$, and components of the quasi-momentum run through the discrete set $k_i = \pi n_i / L_i$ with $n_i = 1, 2, 3, \dots$. For the spherical geometry, the normalized solutions are written as $\phi_{klm}(\mathbf{r}) = \theta(R - r) (2/r)^{1/2} J_{l+1/2}(kr) Y_{lm}(\vartheta, \alpha) / R J_{l+3/2}(kR)$, where $\lambda = (k, l, m)$. Next, the field operators $\hat{\varphi}(x)$ corresponding to the pion field should be expanded

in terms of these eigenfunctions, i.e.

$$\hat{\varphi}(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_{\mathbf{k}}} [a(\mathbf{k})\phi_{\mathbf{k}}(x) + b^+(\mathbf{k})\phi_{\mathbf{k}}^*(x)] , \quad (1)$$

where $a(\mathbf{k})$ and $b(\mathbf{k})$ are the annihilation operators of positive and negative pions, respectively. On the other hand, the states corresponding to confined particles can be written in a common way as $\phi_{\mathbf{k}}(\mathbf{x}) = \sqrt{\rho(\mathbf{x})/V} \Phi_{\mathbf{k}}(\mathbf{x})$, where, for instance, $\sqrt{\rho(\mathbf{x})} = \prod_{i=1}^3 [\theta(L_i - x_i)\theta(x_i)]$ for a box and $\sqrt{\rho(\mathbf{x})} = \theta(R - r)$ for a sphere, respectively. The function $\rho(\mathbf{x})$ represents the information about the geometry of a reaction region or cuts out the volume where the pions (quarks) can annihilate. Hence, for the evaluation of S -matrix elements wave functions $\phi_{\mathbf{k}}(x)$ should be taken as the pion in -states once annihilating pions belong to finite system. The amplitude of the pion-pion annihilation to a lepton pair in the first non-vanishing approximation is calculated via the chain $\pi^+\pi^- \rightarrow \rho \rightarrow \gamma^* \rightarrow \bar{l}l$, where the ρ -meson appears as an intermediate state in accordance with the vector meson dominance. The matrix element of the reaction is $\langle \text{out} | S | \text{in} \rangle = - \int d^4x_1 d^4x_2 \langle \mathbf{p}_+, \mathbf{p}_- | T [H_I^\pi(x_1) H_I^l(x_2)] | \mathbf{k}_1, \mathbf{k}_2 \rangle$, where $H_I^\pi(x) = -e j_\mu^\pi(x) A^\mu(x)$ and $H_I^l(x) = -e j_\mu^l(x) A^\mu(x)$. It is remarkable that the pion density $\rho(\mathbf{x})$ appears as a factor of the pion current. Indeed,

$$j_\mu^\pi(x) = -i\hat{\varphi}(x) \overleftrightarrow{\partial}_\mu \hat{\varphi}^+(x) = \frac{\rho(\mathbf{x})}{V} \left[-i\hat{\Phi}(x) \overleftrightarrow{\partial}_\mu \hat{\Phi}^+(x) \right] , \quad (2)$$

where the field operator $\hat{\Phi}(x)$ is defined in the same way as that in (1) with the functions $\phi_{\mathbf{k}}(\mathbf{x})$ replaced by $\Phi_{\mathbf{k}}(\mathbf{x})$. Because of this factorization, after the integration over the vertex x the density $\rho(\mathbf{x})$ automatically cuts out the volume, where the $\pi^+\pi^-$ annihilation reaction is running. At the same time, this means that the density $\rho(\mathbf{x})$ determines the volume of quantum coherence, i.e. just the particles from this spatial domain are capable to annihilate one with another and make contribution to the amplitude of the reaction. To obtain the overall rate, it is necessary then to sum up the rates from every coherent domain of the fireball.

For the sake of simplicity one can assume that the pion states can be approximately represented as $\phi_{\mathbf{k}}(x) = \sqrt{\rho(x)/V} e^{-ik \cdot x}$ ($\rho(x)$ is the 4-density of pions in the volume V where pions are in a local thermodynamic equilibrium). In essence, this approximation considers just one mode of the wave function $\Phi_{\mathbf{k}}(\mathbf{x})$ and reflects the qualitative features of the pion states in a real hadron plasma. In the frame of this approximation a simple calculation immediately shows that the S -matrix element is proportional to the Fourier-transformed pion density $\rho(x)$, i.e. $\langle \text{out} | S | \text{in} \rangle \propto \rho(k_1 + k_2 - p_+ - p_-)$, where k_1 and k_2 are the 4-quasi-momenta of the initial pion states and p_+ and p_- are the 4-momenta of the outgoing leptons. This means that the form-factor of the pion source $\rho(k)$ stands here in place of the delta function which appears in the standard calculations, i.e. $(2\pi)^4 \delta^4(K - P) \rightarrow \rho(K - P)$, where $K = k_1 + k_2$ and $P = p_+ + p_-$ are the total (quasi-) momenta of pion and lepton pairs, respectively. An immediate consequence of this is a breaking down of the energy-momentum conservation in the s -channel of the reaction, which means

that the total momentum K of the pion pair is no longer equal exactly to the total momentum P of the lepton pair. The physical interpretation of this fact is rather obvious: the effect of the hadron environment on the pion subsystem which prevents the escape of pions from the fireball can be regarded during the time span τ as the influence of an external nonstationary field. The latter, as known, breaks down the energy-momentum conservation. From now, the squared form-factor $|\rho(K - P)|^2$ of the pion system plays the role of a distribution which in some sense "connects" in s-channel the annihilating and outgoing particles instead of δ -function. Indeed, the number $N^{(\rho)}$ of produced lepton pairs from a finite pion system related to an element of the dilepton momentum space, reads

$$\left\langle \frac{dN^{(\rho)}}{d^4P} \right\rangle = \int d^4K |\rho(K - P)|^2 \left\langle \frac{dN}{d^4K d^4P} \right\rangle, \quad (3)$$

where

$$\left\langle \frac{dN}{d^4K d^4P} \right\rangle = \int \frac{d^3k_1}{(2\pi)^3 2E_1} \frac{d^3k_2}{(2\pi)^3 2E_2} \delta^4(k_1 + k_2 - K) f_{\text{BE}}(E_1) f_{\text{BE}}(E_2) \quad (4)$$

$$\times \int \frac{d^3p_+}{(2\pi)^3 2E_+} \frac{d^3p_-}{(2\pi)^3 2E_-} \delta^4(p_+ + p_- - P) |A_0(k_1, k_2; p_+, p_-)|^2. \quad (5)$$

Here, $E_i = \sqrt{m_\pi^2 + \mathbf{k}_i^2}$, $i = 1, 2$ for pions and $E_i = \sqrt{m_l^2 + \mathbf{p}_i^2}$, $i = +, -$ for leptons, respectively. To obtain Eq.(3), we represent the amplitude of the reaction as

$$\langle \text{out} | S^{(2)} | \text{in} \rangle = \rho(k_1 + k_2 - p_+ - p_-) A_0(k_1, k_2; p_+, p_-). \quad (6)$$

Note that not only the form-factor $\rho(K - P)$ contains information about the pion system. The amplitude A_0 carries also new important features, which are related to the violation of the energy-momentum conservation in the s -channel. The latter is a consequence of the medium effects through a partial confinement of the pion states inside fireball what results in the breaking of the translation invariance. Indeed, the pion-pion c.m.s. moves with the velocity $\mathbf{v}_K = \mathbf{K}/K_0$, whereas the lepton-pair c.m.s. moves with the velocity $\mathbf{v}_P = \mathbf{P}/P_0$. Hence, these two center-of-mass systems are "disconnected" now, that is why any quantity should be Lorentz-transformed when transferred from one c.m.s. to another. Reflection of this is the appearance of the correction factor $\left[1 + \frac{1}{3} \left(\frac{(P \cdot K)^2}{P^2 K^2} - 1 \right) \right]$ in the formula for the dilepton production rate (for details see [9, 10]).

By the broken brackets in Eq.(3), we denote the thermal averaging over the pion quasi-momentum space. Actually, we assume a *local thermal equilibrium* in the multi-hadron (-pion) system. Hence, the Green's function, $D^<(x_1, x_2) = \langle \hat{\Phi}^+(x_2) \hat{\Phi}(x_1) \rangle$, which appears after thermal averaging, can be represented as

$$D^<(x_1, x_2) = \int \frac{d^4k}{(2\pi)^4} e^{-ik \cdot x} A(k) f_{\text{BE}}(k, X), \quad (7)$$

where $X = (x_1 + x_2)/2$, $x = x_1 - x_2$, $f_{\text{BE}}(k, X) = \{\exp[\beta(X)(k \cdot u(X) - \mu(X))]\}^{-1}$ is the Bose-Einstein distribution function, which depends on space-time variables, $X = (X^0, \mathbf{X})$, β is the inverse temperature, $u(X)$ is the hydrodynamical velocity and μ is the chemical potential. For the ideal gas (infinite life time of the system) the spectral function $A(k^0, \mathbf{k})$ indicates that all states are on the mass-shell: $A(k) = 4\pi\delta(k^2 - m_\pi^2)\theta(k^0)$. In the interacting system the spectral function reflects a collision broadening of the states which includes as well a global decay of the system if collisions in the system exist during finite time span. For instance, the fireball, which is nothing else as a system of strongly interacting particles, lives until its decay, i.e. starting from the creation till the freeze-out, after which there are no strong interactions between particles at all. In our further consideration we will take into account just a global decay of the multi-pion (multi-quark) system. Assuming a proper model of the spectral function, $A(k)$, which responsible in the present approach for finite life time of the system, we incorporate it together with the spatial density $\rho(\mathbf{x})$ to the global system form-factor $\rho(x)$.

Concerning the physical meaning of Eq. (3), we note that one can regard it as the averaging of the random quantity $\langle \frac{dN}{d^4K d^4P} \rangle$ with the help of the distribution function $|\rho(K - P)|^2$ centered around the value P , which is fixed by experimental measurement. In this sense, the hadron medium holding pions in a local spatial region for some time, which is expressed as the local pion distribution $\rho(x)$, plays the role of an environment randomizing the pion source. This randomization is a purely quantum one in contrast to the thermal randomization of the multi-pion system which is already included to the quantity $\langle \frac{dN}{d^4K d^4P} \rangle$.

3. Dilepton emission rates

In order to transform the distribution of the number of created lepton pairs over the dilepton momentum space to the distribution over invariant masses, one has to perform additional integration using $\langle dN^{(\rho)}/d^4P \rangle$ from (3), i.e. $\langle \frac{dN^{(\rho)}}{dM^2} \rangle = \int \frac{d^3P}{2P_0} \langle \frac{dN^{(\rho)}}{d^4P} \rangle$, where $P_0 = \sqrt{M^2 + \mathbf{P}^2}$. This results in:

$$\begin{aligned} \left\langle \frac{dN^{(\rho)}}{dM^2} \right\rangle &= \frac{\alpha^2}{3(2\pi)^8} \left(1 - \frac{4m_e^2}{M^2}\right)^{1/2} \left(1 + \frac{2m_e^2}{M^2}\right) |F_\pi(M^2)|^2 \int \frac{d^3P}{2P_0} \int d^4K \frac{K^2}{M^2} \\ &\times |\rho(K - P)|^2 e^{-\beta K_0} \left(1 - \frac{4m_\pi^2}{K^2}\right)^{3/2} \left[1 + \frac{1}{3} \left(\frac{(P \cdot K)^2}{M^2 K^2} - 1\right)\right], \end{aligned} \quad (8)$$

where we take the Boltzmann distribution $f_{\text{BE}}(E) \approx \exp(-\beta E)$. Note, that during integration with respect to a 4-momentum K one should keep the invariant mass of a pion pair, $M_\pi = \sqrt{K^2}$, not less than two pion masses. On the other hand, possible finite values of the distribution $\langle \frac{dN^{(\rho)}}{dM^2} \rangle$ below the two-pion mass threshold can occur just due to the presence of the pion system form-factor $\rho(K - P)$. The factor in the square brackets on the r.h.s. of (8) is a correction which is due to the

Lorentz transformation of the quantity $(\mathbf{k}_1 - \mathbf{k}_2)^2$ from the dilepton c.m.s. to the pion-pion c.m.s. This factor gives a remarkable contribution to the dilepton spectrum for invariant masses below the two-pion mass value. Its influence is especially pronounced for e^+e^- production as was shown in [9, 10].

In Eq.(8) the ρ -meson form-factor, $F_\pi(M^2)$, is a vacuum one. Actually, there are two ways to take into account effects of the hadron medium: first, one can account for distortion of the pion states caused by dense environment; second, one can look for ρ -meson polarization effects during its passing through the hadron environment. In the present paper we choose the first way of accounting for the medium effects (see also [11]). Just to elucidate as much as possible the consequences of the contraction of the pion states in the hadron medium and to prevent a double counting we take the vacuum ρ -meson form-factor.

For particular evaluations we take as a model of the pion system the Gaussian distribution of the particles in space and the Gaussian decay of the system (this form-factor succeeded in HBT interferometry):

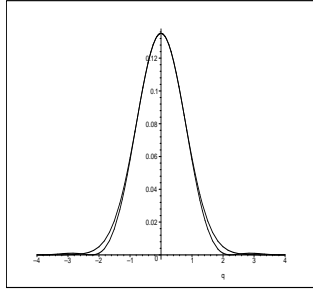


Fig. 1. Comparison of the Gaussian and θ -function form-factors $|\rho(\mathbf{q})|^2/V$.

$$\rho(x) = \exp\left(-\frac{x^2 - 2(u \cdot x)^2}{2R^2}\right), \quad (9)$$

where u is the hydrodynamical (collective) velocity of the element of the total system which is in a local thermodynamic equilibrium; R is the spatial radius of the element. In the rest frame of the element the form-factor looks like: $\rho_0(t, \mathbf{r}) = \exp[-(t^2 + \mathbf{r}^2)/2R^2]$. To get a proper interpretation in terms of the mean life time of the system element, τ , one needs to make a scale transformation during integration over the time variable: $\int_{-\infty}^{\infty} dt \rho_0(t, \mathbf{r}) F(t, \mathbf{r}) = \int_{-\infty}^{\infty} dt \exp(-t^2/2\tau^2 - \mathbf{r}^2/2R^2) F_0(t, \mathbf{r})$, where $F_0(t, \mathbf{r}) = \frac{R}{\tau} F(\frac{R}{\tau}t, \mathbf{r})$.

Meanwhile, it can be another choice of the pion source function. Indeed, one can choose, for instance, a geometry with sharp boundaries which are determined by the θ -function. To show that the final answer is not sensitive to the form of the cutting function we compare two form-factors (normalized to the unit volume) which correspond to the Gaussian distribution $\rho(\mathbf{r}) = \exp(-\mathbf{r}^2/2R^2)$ and to the θ -function distribution $\rho(\mathbf{r}) = \theta(R - |\mathbf{r}|)$ (see Fig. 1). Only a slight difference between these form-factors is seen and, therefore, the choice of pion source distribution does not affect much the dilepton production rate.

We evaluate the rate in the rapidity window, $y_{\min} \leq y \leq y_{\max}$, which corresponds to CERES experimental conditions [1, 2]

$$\frac{dR}{dM dy} = 2\pi M \frac{1}{\Delta y} \int_{y_{\min}}^{y_{\max}} dy \int_{P_{\perp \min}}^{\infty} dP_{\perp} P_{\perp} \frac{dN}{d^4x d^4P},$$

where $\tanh y = P^3/P^0$, $P_\perp^2 = (P^1)^2 + (P^2)^2$. The results of evaluation of the production rates $dR_{e^+e^-}^{(\rho)}/dMdy$ and $dR_{\mu^+\mu^-}^{(\rho)}/dMdy$ for electron-positron and muon-muon pairs, respectively, in pion-pion annihilation are depicted in Fig. 2. Note, the calculations are carried out in the frame of the element of the system where particles are in a local thermal equilibrium. Different curves correspond to the different "spatial sizes" R and different "lifetimes" τ (for particular values of these parameters see Fig. 2) of a hot pion system at the temperature $T = 180$ MeV.

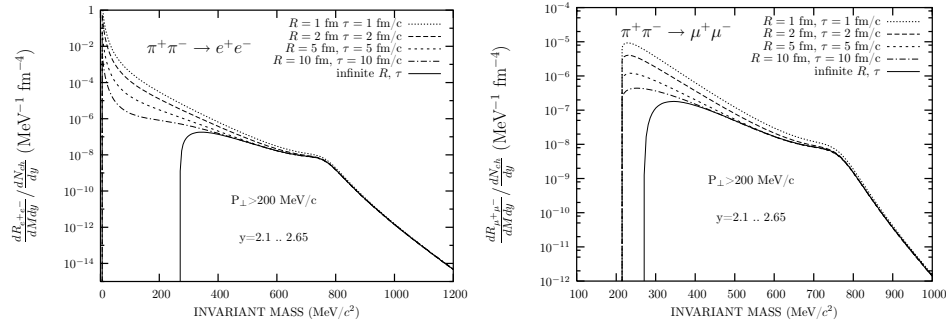


Fig. 2. Rates of electron-positron (left panel) and muon-muon (right panel) productions in pion-pion annihilation in a small finite system, $T = 180$ MeV.

For comparison, we present in Fig. 3 the results of evaluation of the rate $dR_{e^+e^-}^{(\rho)}/dMdy$ of electron-positron pair production in quark-antiquark annihilation

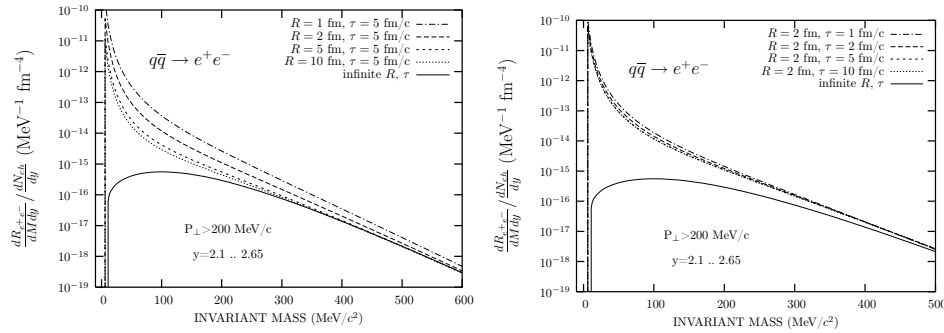


Fig. 3. Electron-positron production rates in quark-antiquark annihilation in a hot QGP drop.

in a hot QGP drop, $T = 180$ MeV. The evaluation was carried out in the frame of the quark drop under the same assumptions as for pion-pion annihilation. As in the previous case, an increase in the rate with decrease in the invariant mass up to two electron masses is seen. This real threshold is close to the total mass of annihilating quarks $M = 2m_q \approx 10$ MeV/c².

4. Conclusions

We notice that the production rate in a finite small system differs from the rate in an infinite pion gas (solid curve) where pion *in*-states can be taken as plane waves. The deviation bigger when the parameters R and τ are smaller. Of course, this is a reflection of the uncertainty principle which is realized by the presence of the distribution $|\rho(K - P)|^2$ as the integrand factor in (3). Basically, the presence of the form-factor of the multi-pion system will result in a broadening of the rate for small invariant masses $M \leq 800 \text{ MeV}/c^2$ which is wider at the smaller parameters R and τ . This seems natural because the quantum fluctuations of the momentum are more pronounced in smaller systems. We emphasize as well that the behavior of the curves in Fig. 2 which correspond to a finite system has a similar to the CERES data tendency [1, 2].

The same behavior of the rate is seen in a hot quark drop (see Fig. 3): for small parameters R and τ in the region of small invariant masses $M \leq 500 \text{ MeV}/c^2$, as compared to the rate for infinite parameters $R = \infty$, $\tau = \infty$, is due to a rise of quantum fluctuations which are evidently bigger for a smaller size of the QGP drop.

Note that the enhancement of the dilepton production rate for the low invariant mass region is much more sensitive to the variation in the spatial size of a many-particle (pion, quark) system than to that in the system lifetime (see Fig. 3, right panel) [12].

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